CONFLICT RESOLUTION. RISK-AS-FEELINGS HYPOTHESIS.

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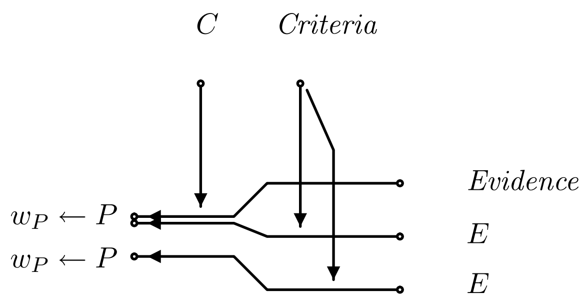
Abstract. Mathematical model of a goal-oriented thinking with feedback is described. Basic notions: *decision graph, feedback hierarchy and self-duality* are introduced and explained. A source of the conflict in our approach is the ignorance about the importance of decision maker's goals. In contrast to Shafir, Simonson & Tversky [4] and Deutsch [2] conflict resolution is modeled as a problem of finding a fixed point of a self-assessment operator, i.e. without adding or removing any decision element from decision hierarchy.

1. Introduction

A subject of this paper is a mathematical model of a well known model of human thinking called ***goal-oriented thinking***, see Baron (1994) [1] for example. A simple example of such model is when two, or more, possibilities (P) lead towards the realization of a goal. When

Figure 1. Goal-oriented thinking. w_P represents the weight of the possibility to be determined.



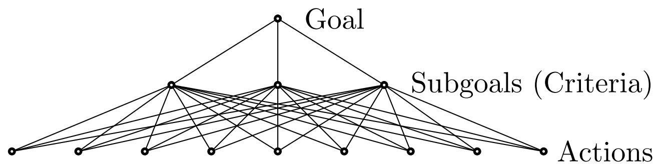
someone is indecisive about his best option, the thinking process

Key words and phrases. multi-criteria decision making, duality, self-ranking, preference graph, potential method, risk-as-feeling model.

begins. In goal-oriented thinking a human mind needs Evidence (E) supported by Criteria (C) to conclude which Possibility (P) is the best one.

Goal-oriented thinking can be represented as a hierarchy, with possibilities, also called ***alternatives*** or ***actions***, at the bottom of the hierarchy. The main goal is on the top of the hierarchy, followed by ***sub-goals***, ***criteria***,.... Figure 2 shows a typical hierarchical decision model. Output of a decision process is the ***ranking*** of the alternatives, i.e. a numerical value function on the set of alternatives. It is often presented as an ordered list where the most preferable alternative appears at the top of the list. Generally speaking, the

FIGURE 2. Hierarchy of a Goal-oriented thinking.



ranking procedure starts from the Goal as a criterion with rank 1. For each element of an already ranked level, as a criterion, the elements of a lower level are ranked. Repeating this process we rank the bottom level.

The hierarchy structure described above is a model for many decision problems mainly used in Analytical Hierarchy Process (AHP) proposed by Saaty [8]. Ranking methods used in AHP can be: Eigenvalue Method (EVM), Geometric Mean Method (GMM) (Saaty and Vargas [9]), Potential Method (PM) (Čaklović [15]). PM uses preference graph rather then reciprocal matrix to capture input data, and can operate with incomplete data as well, which is not the case with EVM and GMM method.

Organization of the paper is the following. The meaning of a notion *conflict* is discussed and explained in Section 2 (What is conflict). In section 3 (Measurement process) we explain a procedure of gathering and organization of input data in Potential Method. A notion of *self-duality* is crucial in our model; its explanation is given in Section 4 (Duality and self-duality). A conflict resolution, Section 5 (Conflict resolution), is described as an iterative self-assessment procedure with unknown weights. The role of emotions in choice under risk is described by *risk-as-feelings* hypothesis (Loewenstein

& others (2001) [6]). The Section 6 (Choice under risk) describes a mathematical model of the risk-as-feelings hypothesis as a decision hierarchy with feed back. According to our knowledge, this is the first mathematical model of this hypothesis in the literature. The Appendix explains in details the aggregation procedure in PM. Existence of the fixed point of the self-aggregation operator is a consequence of a contraction principle. The proof can be found in [12].

2. What is conflict

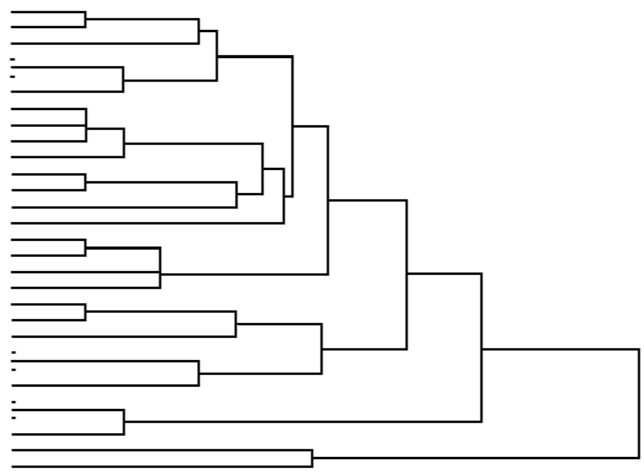
The need to choose often creates conflict: we are not sure how to *trade off one attribute relative to another* or, for that matter, *which attributes matter to us most*. It is a common place that we often attempt to resolve such conflict by seeking reasons for choosing one option over another. At times, the conflict between available actions is hard to resolve, which may lead us to seek additional options, or to maintain the status quo. Shafir, Simonson & Tversky [4] consider the role of reasons and arguments in making decisions. They propose to decision makers, when faced with the need to choose, to seek and construct reasons in order to resolve the conflict and justify their choice. Seeking an additional options in criteria–alternative context means adding (removing) an object to (from) the hierarchy structure. This leads to another decision problem which, from the mathematical point of view, does not create extra difficulties.

It often happens that people *have not well-established values*, and that preferences are actually constructed, not merely revealed, during their elicitation (cf. Payne, Bettman, & Johnson [7]). Different frames, contexts, and elicitation procedures highlight different aspects of the options and bring forth different reasons and considerations that influence decision. From the hierarchical point of view this means that decision maker can change the importance of his goals or sub-goals or change the preferences between the elements in the hierarchy, which leads again to a new decision problem.

According to Deutsch [2], conflict exists whenever *incompatible activities occur*. An activity that is incompatible with another is one that prevents, blocks, or interferes with the occurrence or effectiveness of the second activity. A conflict can be as small as a disagreement or as large as a war. It can originate in one person, between two or more people, or between two or more groups.

Size of the conflict between two or more people can be measured by measuring dissimilarities between their preferences. To be more precise, let us suppose that two or more decision makers gave their

FIGURE 3. Cluster analysis of a group — dendogram.



individual preference graphs[1] over the same set of alternatives. It is difficult (if not impossible) to define a graph distance in general, but in decision making context it can be defined as the distance of the ranking functions generated by those graphs, Čaklović [13]. A distance matrix for the whole group of decision makers can be now calculated and group clasterization can be performed. The number of clasters, and their distances, may indicate the hidden conflict inside the group, according to the given set of criteria. The Figure 3 shows the dendogram of the cluster analysis of a group of 29 students which gave their preferences on the set of their lecturers[2]. The bottom cluster, with 2 elements, is recognized as an out layer. We found out that one student intentionally gave bad points (two times) to one of the lecturers in consideration.

Measuring the conflict does not solve it. In this article we suppose that decision maker exhausted all possibilities to add other option into consideration, i.e. that he ***does not change the structure*** of the hierarchy and ***does not change the preferences*** between the objects inside the hierarchy. A ***source of the conflict*** are ***unknown weights (importance)*** of his goals. Next step in conflict resolution is described in Section 5.

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[1]Explanation of the *preference graph* is given in the Appendix.

[2]at the Mathematical Department, University of Zagreb.

3. Measurement process

In measuring weight, height, wellness... we suppose that the objects being measured are separated from the measuring engine and are sharing a common measured quality. The result of the measuring process is a number or some categorical value. The following statement is accepted as true:

> **Axiom 3.1.** Two objects are considered different if they have a common quality with different intensity.

Axiom 3.1 is accepted as being true in every measurement theory. If two objects have no common qualities we are not able to compare them and we consider them incomparable.

In everyday decisions, objects and different scenarios are compared, usually in pairs, with respect to certain (attributes) criteria. Moreover, the intensity of that preference is given to each pair of compared objects. In Potential Method, for each criterion, decision maker creates a preference graph on the set of all alternatives. Incomparable nodes are not connected. Orientation of the arc is towards the more preferred node[3] and the intensity of the preference equals zero for equally preferred nodes. A non-negative function $\mathcal{F}$ which associate to each pair of compared nodes the intensity of the preference is called a ***preference flow***. In group (or multicriteria) decision a suitable aggregation procedures gives an overall preference graph (flow) which may be used to calculated the overall ranks of the alternatives. Details can be found in Čaklović [13], [14] and in Appendix.

Human mind makes creates preferences in two ways: subconsciously and consciously. In urgent situations such as panic or fear an immediate response is needed and this is done subconsciously by amygdala and other centers in the brain. On the conscious level decision process takes place in the cortex and it is time consuming. Intensity of the preference is determined by past experiences and emotions. In neuroscience it is well known that the relation cortex-amygdala is extremely important in making decisions together with emotional landscape of the whole body (LeDoux [5]).

4. Duality and self-duality

The first association with the word ***duality*** is 'more than one'. Let us see what some dictionaries tell about duality:

[3] In case of equally preferred nodes arc orientation is arbitrary.

***Duality (Webster dictionary)*:** A grammatical number category referring to two items or units as opposed to one item (singular);

***Double, dual, twofold (answers.com)*:** Having more than one decidedly dissimilar aspects or qualities.

Apart from those literary definitions of duality its ***mathematical concept*** is necessary. According to axiom 3.1 measuring leads to duality. Dual objects are functions defined on the set of objects being measured. In geometry, for example, a dual object is a function that calculates coordinate of the point (projection).

In decision making context ***dual objects are criteria***, we may think of them as observed qualities with appropriate measurement scales, and primal objects are those alternatives which are compared with respect to common criteria. In hierarchical presentation of a decision problem dual objects are in the level(s) above the level under consideration (Figure 2).

Self-duality in decision process arises when some objects are also criteria for themselves. A typical example of self-duality is a group of decision makers which attempt to rank themselves. Each group member creates his own preference graph over the set of all group members including himself. Those preference graphs are then conjoined in one unique graph and the group ranks may be calculated.

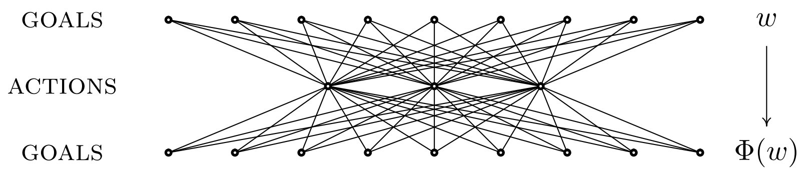
In goals-actions context, self-duality appears when the goals are measured from the point of view of actions.

$$\text{GOALS} \longrightarrow \text{ACTIONS} \longrightarrow \text{GOALS}.$$

To clarify this idea let us fix some weights w of the goals. Roughly speaking, decision maker can try to find, for each action, the goal which is most favorable for that action and 'raise its importance'. From the point of view of another action, some other goal may be more favorable and so on. . . This idea is used in SMART method for determining criteria weights (swing) Edwards-Barron (1994) [3].

More precisely, decision maker creates, for each action, a preference graph on the set of goals from the point of view of that action. Technically speaking, the hierarchical structure is expanded by one or more levels in such a way that the goals appears (again) at the bottom of the expanded hierarchy (Figure 4). The result is a ***hierarchy with feedback*** i.e. the hierarchy with the same object in the first and in the last level. For each initial ranking w of the goals (in the first level) we calculate the new ranks $\Phi(w)$ of the goals (in the bottom level) and repeat the process. A fixed point of the function Φ we interpret as the ranking of the goals which was not known a priori. The details are given in the Section 5. In group self ranking

FIGURE 4. Self-dual hierarchy.



there is no intermediate level (actions) between the self ranking level (goals).

5. Conflict resolution

In the previously described process, the change of goals' weights w directly influences the weights of actions and indirectly changes the weights $\Phi(w)$ of the goals because of the feedback. Intuitively, it is clear that 'real' weights w should be stable on the transformation $w \mapsto \Phi(w)$. In mental and real-life experiments, the mind is trying to adjust the weights w and find the stability in the above sense. If we repeat the above process the infinite sequence of weights is obtained:

$$w \mapsto \Phi(w) \mapsto \Phi(\Phi(w)) \mapsto \cdots \mapsto \Phi^n(w) \mapsto \cdots \tag{5.1}$$

We shall prove that this sequence has a unique fixed point[4] λ, i.e. the point which satisfies the equation

$$\lambda = \Phi(\lambda).$$

Moreover, λ is independent of the first choice of w, cf. Appendix, Theorem 7.1.

6. Choice under risk

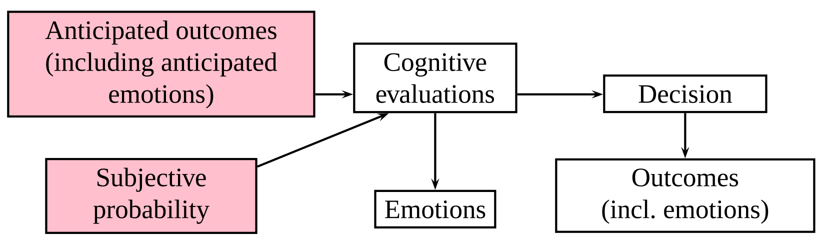
In this example we want to show that self ranking procedure can be applied to *risk-as-feeling* model of choice under risk or uncertainty. The divergence between emotional reactions to risk (and cognitive evaluations of) is a common source of the feeling of inter personal

[4]With some minor restrictions on the number of nodes in the graph.

conflict (see, e.g., Schelling, 1984 [10]). Psychologists from different sub disciplines have been drawing similar distinctions between two qualitatively different modes of information processing. Sloman (1996) [11], for example, makes distinction between rule based and associative processing.

6.1. **Traditional model.** Loewenstein (2001) distinguish *anticipated* and *anticipatory* emotions. He argue that in decision making under risk, emotions inform decision making and, on the other side, emotional responses (anticipatory emotions) to risky situations often diverge from cognitive evaluations. From the consequentialist per-

FIGURE 5. Traditional (consequentialist) model.



spective (which include anticipated emotions) decision makers are assumed to anticipate how they will feel about obtaining different outcomes. Hierarchical structure of this model has three levels:

decision hierarchy
desirability (goal)
outcomes
actions

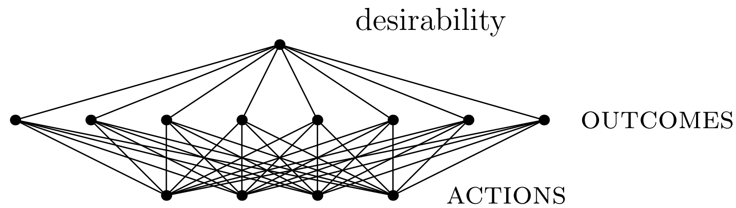
goal-level with *desirability* as the main goal, *outcomes* and *actions*.

The hierarchical structure is given in figure 5. In the first step pairwise comparisons of outcomes[5] are done with respect to the main goal (desirability). In the second step we make pairwise comparisons of actions with respect to each outcome, giving the priority to the action which affect (maintain) the outcome in consideration with

[5]Anticipated emotions are a component of the expected consequences of the decision; they are emotions that are expected to occur when outcomes are experienced, rather than emotions that are experienced at the time of decision.

greater probability[6]. Aggregated preference graph for actions may be used now to calculate their weights.

FIGURE 6. Hierarchy of the consequentialist model.



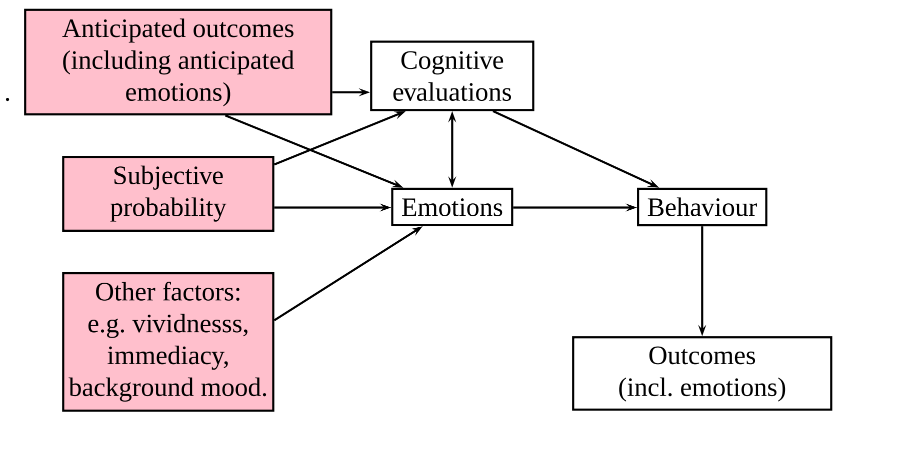
6.2. **Risk-as-feelings model.** Risk-as-feelings hypothesis is illustrated in Figure 7. Loewenstein & others (2001) [6] argue that in risk-as-feelings model...

> ...people are assumed to evaluate risky alternatives at a cognitive level, as in traditional models, based largely on the probability and desirability of associated consequences. Such cognitive evaluations have affective consequences, and feeling states also exert a reciprocal influence on cognitive evaluations. At the same time, however, feeling states are postulated to respond to factors, such as the immediacy of a risk, that do not enter into cognitive evaluations of the risk and also respond to probabilities and outcome values in a fashion that is different from the way in which these variables enter into cognitive evaluations.

One possible self-dual hierarchical structure of the risk-as-feeling model is given in Figure 8. (1) In ***the first step*** we compare outcomes with respect to one criterion, *risk* for example. We give the priority to the outcome which is less risky. (2) In ***the second step*** we compare actions with respect to each outcome in the same way as in traditional model. (3) In ***the third step*** we compare outcomes with respect to actions. For the fixed action we give the priority to the outcome (among two of them) which has greater probability to happen. (4) In ***the forth step*** we compare criteria *risk* and *desirability* with respect to outcomes. The question is which criterion (as a principle) is sustained more by given outcome.

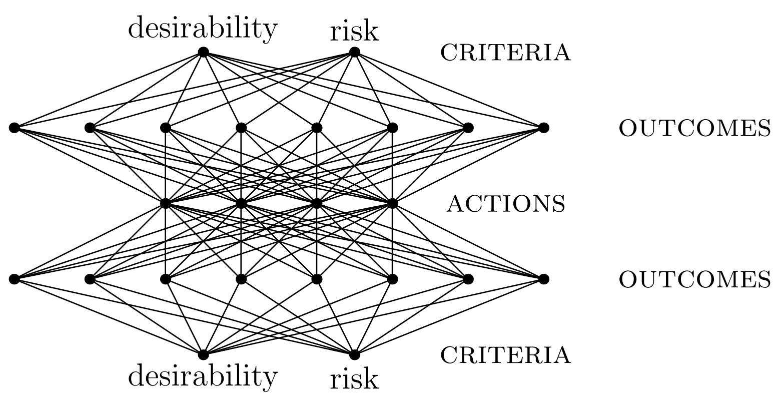
[6]May be the 'probability' is not the best word to use it here but we do not know the better one. We are using it because Loewenstein & others (2001) use the same notion.

FIGURE 7. Risk-as-feelings perspective.



In the risk-as-feelings hierarchical model no a priori weights are given. We have only a bounce of preference graphs which may be aggregated into the group preference matrix. Theorem 7.1 allows us

FIGURE 8. Self-dual hierarchy of the risk-as-feeling model.



to calculate the fixed point[7] of Φ which we interpret as the ranking of decision elements in the first level. The ranks of the elements in

[7]Convergence of the iterative process 5.1 can may be made faster if the hierarchy is rearranged in such a way that the level with more nodes become the first level. This is clear from the proof of Theorem 7.1 and at this moment is out of

other levels can be calculated now in the same way as in the classical hierarchy.

6.3. **An example. Choice of the climbing route.** In this example two climbers have intention to climb in the Dolomites, north Italy. Their possible choices (after a long discussion) of the climbing

TABLE 6.1. Consequentialist approach – levels:

desirability→outcomes→actions
criteria: desirability (goal)
outcomes: bivouac, exitVF, glory, effort
actions: via ferrata, classical route, first repetition

routes are (one of): *via ferrata*[8], *classical route*[9] and *first repetition*[10]. Possible outcomes are: spending a night in *bivouac* sac (if time goes

TABLE 6.2. Risk-as-feelings approach – levels:

actions→outcomes→criteria→outcomes→**actions**
outcomes: bivouac, exitVF, glory, effort
criteria: risk, desirability
actions: via ferrata, classical route, first repetition

the scope of this article. This rearrangement of the levels seems to be natural because of the feed back incorporated in the problem.

[8] *Via ferrata* is italian name for 'road with irons', a mountain route which is equipped with fixed cables, stemples, ladders, and bridges. The use of these allows otherwise isolated routes to be joined to create longer routes which are accessible to people with a wide range of climbing abilities. Walkers and climbers can follow *vie ferrate* without needing to use their own ropes and belays, and without the risks associated with unprotected scrambling and climbing.

[9] Classical (or traditional) route is well documented in the climbing guidebooks, sometimes well equipped at anchor points. It involves also the placement of temporary protection such as cams, nuts, and hexes, into the rock's natural features while ascending.

[10] The first and the second repetition of a new climbing route is usually noted in the climbing literature. Sometimes, the first repetition may be as difficult as the first ascent of the route. Documentation given by the first climbers is often available (climbing time, descent, climbing grades, sketch of the route. . .)

slowly), *exit* through via ferrata (in case of emergency), *glory* (after finishing the first repetition) and *effort*.

Traditional (consequentialist) model has three levels in the hierarchy as shown in the Figure 6.1, while risk-as-feelings model has two levels more, see the Figure 6.2. Some extra explanation is needed at this point. In a self-dual hierarchy the first level can be any level in the hierarchy. On the other side, the Theorem 7.1 gives that the convergence of the iterative process $w \mapsto \Phi^n(w)$ is faster in higher dimensions, i.e. when the number of nodes in the first level is greater. Because of that we choose the level *actions* to be the first one.

Solutions for both decision problems are given in the Table 6.3. We can see from the table that rankings are different, although the

TABLE 6.3. Ranking of climbing routes. Both methods.

Ranking results			
	climbing route		
method	via ferrata	classical route	first repetition
traditional	0.057	0.126	0.818
risk-as-f	0.218	0.199	0.583

common preferences inside both hierarchy are the same. The interpretation is left to the reader. May be the emotional feed back which influence the comparisons in the step (3) and (4) (page 9) changes the whole feed back dynamics. From the Table 6.3 we see that the fixed point of the iteration process is obtained in 9 iterations (precision $\varepsilon = 0.0001$). In the first step the ranking is still the same as in traditional model, while in the second step the ranking is already the same as in the fixed point. For testing numerical procedure

Fixed point			
step	via ferrata	classical route	first repetition
0.	1/3	1/3	1/3
1.	0.173	0.188	0.639
2.	0.234	0.202	0.565
3.	0.213	0.198	0.589
...	...	...	...
9.	0.218	0.199	0.583

please visit URL http://decision.math.hr/examples/ and find a link *Conflict resolution.*

7. Appendix

A ***preference graph*** is a digraph $\mathcal{G} = (V, \mathcal{A})$ where V is the set of nodes and $\mathcal{A}$ is the set of arcs of $\mathcal{G}$. The set of arcs $\mathcal{A}$ is an antisymmetric relation on V. We say that node a is ***more preferred*** than node b, in notation $a \succcurlyeq b$, if there is an arc $(a, b) \in \mathcal{A}$ outgoing from b and ingoing to a. A ***preference flow*** is a non-negative real function $\mathcal{F}$ defined on the set of arcs. The value $\mathcal{F}_\alpha$ on the arc α is an intensity of the preference α on some scale[11]. $\mathcal{F}_\alpha = 0$ means that the decision maker is indifferent for the pair $\{a, b\}$. In that case orientation of the arc is arbitrary.

A preference flow is ***consistent*** if there is no component of the flow in the cycle-space of the preference graph. It is easy to see that the following statements are equivalent:

(1) $\mathcal{F}$ is consistent.
(2) The sum of algebraic components of the flow along each cycle is equal to zero.
(3) $\mathcal{F}$ is a linear combination of the columns of the incidence matrix A of the preference graph.
(4) There exists $X \in \mathbb{R}^n$ such that $AX = \mathcal{F}$.
(5) The scalar product $y^\tau \mathcal{F} = 0$ for each cycle y, i.e. $\mathcal{F}$ is orthogonal to the cycle space.

We shall examine the consistency of a given flow $\mathcal{F}$ by solving the equation

$$AX = \mathcal{F}. \tag{7.1}$$

In practice, while performing pairwise comparisons, a decision maker does not give a flow which is necessarily consistent, specially if they are subjective. In that case, the best approximation of the flow by the column space of the incidence matrix (the space of consistent flows) should be calculated. ***Potential*** X associated to $\mathcal{F}$ is a solution of the equation (7.1) or a solution the normal equation (7.2) and equation (7.3)

$$A^\tau AX = A^\tau \mathcal{F}, \tag{7.2}$$

$$\sum_{v \in V} X(v) = 0 \quad \text{(uniqueness condition)} \tag{7.3}$$

[11] For subjective pairwise comparisons the scale is $\{0, 1, 2, 3, 4\}$. It is additive, not the multiplicative scale like in AHP.

if the first one doesn't exists. To obtain the ranking from X, the following formula is used

$$w = \frac{a^X}{\|a^X\|_1}, \quad a > 1,$$

where exponential function of X is defined component wise, i.e. $(a^X)_i = a^{X_i}$, where $\|\cdot\|_1$ is 1-norm. Parameter $a > 1$ can be arbitrary. When the preference graph is complete it is easy to show that the equation (7.2) has a solution in the explicit form

$$X_i = \frac{1}{n}\sum_{i=1}^{n}(A^\tau \mathcal{F})_i.$$

Aggregation (we call it consensus) of the flows in case of more than one criterion is done in the following way. Each criterion $C_i, i \in \{1, \ldots, k\}$ generates its own preference graph $(V, \mathcal{A}_i)$ and its own preference flow $\mathcal{F}_i$. Let w_i denotes the weight of i-th criterion. For a given pair $\alpha = (u, v)$ of alternatives we calculate

$$F_\alpha := \sum_{\substack{i=1 \\ \pm\alpha\in\mathcal{A}_i}}^{k} w_i\mathcal{F}_i(\alpha) \tag{7.4}$$

where the item $w_i\mathcal{F}_i(\alpha)$ is taken into account if and only if $\alpha \in \mathcal{A}_i$ or $-\alpha \in \mathcal{A}_i$. If this sum is non-negative, then we include α in the set $\mathcal{A}$ of arcs of the ***consensus graph***, and we put $\mathcal{F}(\alpha) := F_\alpha$ where $\mathcal{F}$ denotes ***consensus flow***. If the sum is negative, we define $-\alpha = (v, u)$ as an arc in $\mathcal{A}$ and $\mathcal{F}(-\alpha) := -F_\alpha$. The flow $\mathcal{F}$ is now well defined. If F_α is not defined then u and v are not adjacent in the consensus graph[12].

A powerful feature of Potential Method is self-ranking. By self-ranking we mean a decision model in which criteria and alternatives are the same. This situation happens in a self-evaluation of a group of decision makers when each member gives a preference graph on the set of all group members. A member can include himself in a preference graph or not. Moreover each group member can use its own criteria in ranking the others.

Let us denote by $\mathcal{G} = \{1, 2, \ldots, n\}$ a group of decision makers and by $\Sigma = \{\xi \mid \sum_{i=1}^{n} \xi_i = 1, \, \xi_i \geq 0\}$ the standard simplex in $\mathbb{R}^n$. Let us suppose that each decision maker $i \in \mathcal{G}$ gave the preference flow $\mathcal{F}_i$

[12]In AHP (eigenvalue approach) consensus is not done naturally; by making convex combinations of the eigenvectors. The consensus should be done over primitive objects i.e. over graphs, like here, or over reciprocal matrices, like in geometric mean approach of AHP.

on the set $\mathcal{G}$ with respect to certain criteria. If $\xi \in \Sigma$ is an a priori given group ranking and $\mathcal{F}_G$ is the consensus flow, then it should satisfy the equation

$$\sum_{i\in\mathcal{G}} \xi_i \, \mathcal{F}_i = \mathcal{F}_{\mathcal{G}} \, . \tag{7.5}$$

Because of linearity of the equation (7.2) the same relation should take place for potentials X_i and the potential $X_{\mathcal{G}}$ of the consensus flow, i.e.

$$\sum_{g\in\mathcal{G}} \xi_i X_i = X_{\mathcal{G}}. \tag{7.6}$$

To simplify the notation let us denote by X the matrix with columns $X_i, \; i = 1, \ldots, n$. Then, the left side of the above equation can be written as a product $X\xi$ between X and the column ξ. A function

$$\Phi : \xi \mapsto \frac{a^{X\xi}}{\|a^{X\xi}\|_1} \tag{7.7}$$

defined on the standard simplex Σ to itself is now well defined and the group ranking derived from the consensus flow should be a fixed point of Φ, i.e. it should satisfy the equation

$$\xi = \Phi(\xi). \tag{7.8}$$

Existence of the fixed point is a consequence of Brouwer's fixed point theorem and the uniqueness is given by the following theorem:

Teorem 7.1. *Let us suppose that*

$$2 \ln a \|X\|_\infty < 1 \tag{7.9}$$

Then, Φ is a contraction and for each $\xi \in \Sigma$ the sequence $\Phi^n(\xi)$ converges to the unique fixed point $\xi_0 \in \Sigma$ of Φ.

The proof can be found in Čaklović [12] (working paper).

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