POTENTIAL METHOD APPLIED ON EXACT DATA

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Abstract. Although PM was basically created to treat subjective comparisons, given exact values can be converted to Potential Method flow. A new feature of PM is a parameter called $FlowNorm$ which helps to treat problems where each criterion has its own measurement scale.

Complex hierarchial structures and problems modelled by decision tables can be handled. Incomplete data, such as incomplete belief, is considered as well. To illustrate the flexibility and universality of PM we compare it with PROMETHEE and Yang's Evidential Reasoning method.

1. Introduction

Although Potential Method (PM) [Ca] was originally created for subjective decision making, it can be applied to problems with exact data. This data can be result of some measurement process, testing procedure or possible costs in future investment. Generally speaking, there are two main approaches in decision theory: those based on decision table and those based on hierarchy. In both of them we shall compare PM with other well-known decision making methods, PROMETHEE and Yang's Evidential Reasoning method (ER) developed in [Ya].

In Section 2 we shall prove equivalence of PM and expected utility theory for decision table with prescribed utility. Some subjective measurements, such as *belief structured* problems (Section 3) can be also treated as exact data type problems. More complex exact data type of problem when each criterion has its own measurement unit, is treated in Section 5. A new parameter, $FlowNorm$ is introduced to treat such problems. In this section we are treating a simple example of marketing problem, Table 5, to compare PROMETHEE method and PM.

Incomplete belief structure is considered in Section 4. This theme is out of the scope of this article and we are giving only a rough idea what can be done.

Two kinds of hierarchical structures — cars assessment and motorcycle assessment — are considered in Section 6. Cars assessment example is combination of exact data and belief structure while motorcycle assessment introduces extra difficulty because of the missing data. PM is compared with Yang's ER.

Examples are chosen from existing literature and documentation that accompany software distribution. Comparative results given in Tables 6 and 10 show the flexibility and universality of PM.

2. Equivalence of Expected utility and Normal Integral

In this section we shall prove that PM is equivalent to Expected Utility approach.

2.1. **Expected utility.** Table 1 shows standard decision table. We have n states of nature or circumstances θ_j, which can be seen as criterions, and m actions a_i, which can be seen as alternatives. Each state has probability P_j. Numbers v_{ij} represent payoff for action a_i taken under the circumstance θ_j.

Key words and phrases. analytical hierarchy process, multi-criteria decision problem, decision, decision table, exact values.

Expected utility theory defines utility of action a_i as

$$U(a_i) := \sum_j P(\theta_j) v_{ij}$$

and maximizes it over the set of all actions to choose the 'best action'.

		States of nature			
		θ_1	θ_2	$\cdots$	θ_n
Actions	a_1	v_{11}	v_{12}	$\cdots$	v_{1n}
	a_2	v_{21}	v_{21}	$\cdots$	v_{21}
	.	.	.	$\cdots$	.
	a_m	v_{m1}	v_{m2}	$\cdots$	v_{mn}

TABLE 1. Decision table.

2.2. **Consensus flow for decision table.** Here is the short description of consensus flow for several criteria.

If one criterion is present then for a given preference flow F, and incidence matrix B of the underlying graph *normal integral* is given as a solution of equations

$$B^\tau B X = B^\tau F, \quad \sum_{i=1}^{m} X_i = 0 \tag{1}$$

If the graph is not connected, normal integral is unique on each connected component of the graph. For more details see [Ca].

If more than one criterion is present, the procedure of making a *consensus graph* $(V, \mathcal{A})$ and *consensus flow* F is as follows. Each criterion C_i generates its own graph $(V, \mathcal{A}_i)$ and its own flow F_i. Let us denote the weight of i-th criterion by w_i, where $\sum_i w_i = 1$.

First, for a given pair $\alpha = (u, v)$ we calculate

$$F_\alpha := \sum_{\substack{i=1 \\ \pm\alpha\in\mathcal{A}_i}}^{k} w_i F_i(\alpha) \tag{2}$$

where the summand $w_i F_i(\alpha)$ is taken in account if and only if $\pm\alpha \in \mathcal{A}_i$. If this sum is non-negative, then we put α in the set $\mathcal{A}$ and $F(\alpha) := F_\alpha$. Otherwise, we define $-\alpha = (v, u)$ as an arc in $\mathcal{A}$ and $F(-\alpha) := -F_\alpha$. The flow F becomes a non-negative flow which we call *consensus flow.*

For decision table (such as Table 1) the consensus flow defined on the graph with actions as vertices, according to formula (2) is given by

$$F_{jk} = \sum_i P(\theta_i)(v_{ki} - v_{ji}), \quad k, j = 1, \ldots, m. \tag{3}$$

Note that such flow is complete.

2.3. **Equivalence.** The following theorem gives motivation for such definition by proving equivalence of PM and expected utility theory in case of exact data given by Table 1.

Theorem 1. *Ranking over the set of alternatives given by expected utility is the same as the ranking given by Potential Method. More precisely*

$$X_k \geq X_l \iff U_k \geq U_l,$$

where X is normal integral on the set of alternatives.

Proof. To prove the theorem, it suffices to show that

$$X_k - X_l = U_k - U_l. \tag{4}$$

To prove that let us calculate normal integral X_k. As the flow F is complete we can calculate X_k as difference between incoming and outgoing flow at vertex a_k divided by number of vertices. Consensus flow can be written, using formula (3), as

$$F_{jk} = \sum_i P(\theta_i)v_{ki} - \sum_i P(\theta_i)x_{ji} = U_k - U_j.$$

which implies

$$X_k = \frac{1}{m}\sum_{j\neq k} F_{jk} = \frac{1}{m}\sum_j (U_k - U_j) = U_k - \frac{1}{m}\sum_j U_j$$

and formula (4) follows immediately. □

3. Belief structure

A belief structure represents an expectation that was originally designed to model a subjective assessment with uncertainty. To evaluate the quietness of an engine (e.g., Honda engine), for example, an expert may state that he is 50% sure it is good and 30% sure it is excellent. In the statement, good and excellent denote distinctive evaluation grades (standards), and the percentage values of 50 and 30 are referred to as degrees of belief, which indicate the extents that the corresponding grades are assessed to.

Generally speaking, belief structure involves multiple attributes (criteria) common for the set of alternatives. Each alternative can be described by *belief function* $v : \mathcal{C} \to [0, 1]$ defined on the set of criteria $\mathcal{C}$ with values in the interval $[0, 1]$ and such that the sum of its values is less than or equal to 1. Values of belief function are called *degrees of belief.* If the sum of all degrees of belief is less than 1 we are talking about *incomplete belief structure.*

Belief structure can be represented as decision table (see Table 1) such that $\sum_{j=1}^{n} v_{ij} \leq 1$.
Honda engine, mentioned above, has belief structure shown in Table 2. This is an example of incomplete belief because the sum of degrees of belief is strictly less than 1.

Degree	Poor	Average	Good	Excellent	Top
quietness	0	0	0	0.5	0.3

Table 2. Incomplete belief structure.

4. Incomplete belief structure

Treating incomplete belief structure can be twofold: to reconstruct data using some heuristic methods or take it as it is (PM will generate incomplete graph). In the second case problem can arise if 'too much data' is missing, namely graph can be unconnected and we will obtain ranking by components.

If some data is missing one can try to recover it if some additional information about probability distribution is available. Fuzzy logic can help too but this is far from the scope of this article.

In motorcycle assessment example, Section 6.2, we tried with homogeneous distribution with the same results as without any other heuristic method. We believe the reason is that only small amount of data is missing.

5. Complex exact data structure

5.1. **Normalization.** Having the exact data doesn't mean that subjectivity is excluded from the model. In fact, it gets introduced on a whole new level. In a classical Potential Method approach we were working with differences between exact values (as flow components) which could be small when compared with absolute values of given data. If this is the case, the decision maker could consider given data equally important in his consideration. Such situation is also one of the reasons for introducing utility in rational decision making. Moreover, some criteria may represent income which should be maximized and some criteria may represent outcome which should be minimized.

For example, let us consider 'Fuel economy' criterion in car assessment (see Table 7) with values 20, 20, 21, 20, 19, 20 for six given cars. From the point of view of 'Fuel economy' as criterion someone might conclude that those cars are equally valued. On the other side, the flow generated by those values gives the following ranks which someone can interpret as too great distinction between the cars. To overcome this difficulty we are introducing a parameter called *FlowNorm* which simply normalizes the flow vector in max-norm to the prescribed value. This concept allows decision maker to increase or decrease a difference between ranks without changing the original input. Evidently, setting the $FlowNorm$ to value 0 gives equal ranks as a consequence.

Cars	Car1	Car2	Car3	Car4	Car5	Car6
Fuel Economy (gpm)	20	20	21	20	19	20
Rank	0.154	0.154	0.308	0.154	0.077	0.154
Rank ($FlowNorm = 1$)	0.163	0.163	0.231	0.163	0.116	0.163

Table 3. Motivation for introducing Flow Norm.

Values calculated for $FlowNorm = 1$ are given in the bottom row. As we can see, the rank remained unchanged, as well as relative differences between cars. However, the absolute differences have shrank. Later, we shall apply this rank in the Car assessment problem (chapter 6.1).

To better understand this situation we calculated the ranks for several values of $FlowNorm$ for criterion 'Acceleration' in cars assessment example, Table 4. This is an illustrative example how $FlowNorm$ should be used.

Identical effect as this one obtained by changing $FlowNorm$ can be achieved by changing the weights of criteria but the first approach doesn't interfere with the original input, making the change transparent. One can see the $FlowNorm$ as unification parameter for different measure units accompanied with each criterion.

	Acceleration						
FlowNorm	-0.25	-1	-2	-4	-10	-50	-61
Car 3	0.178	0.214	0.262	0.360	0.626	0.998	1.000
Car 6	0.173	0.188	0.204	0.218	0.177	0.002	0.000
Car 2	0.170	0.177	0.180	0.169	0.094	0.000	0.000
Car 5	0.170	0.177	0.180	0.169	0.094	0.000	0.000
Car 4	0.160	0.137	0.109	0.062	0.008	0.000	0.000
Car 1	0.150	0.107	0.066	0.023	0.001	0.000	0.000

TABLE 4. Effects of *FlowNorm* change.

To conclude, we suggest decision maker to try several values of *FlowNorm* for each criterion before making consensus flow in multi-criteria case.

5.2. **Marketing problem.** In the recent years several decision aid methods or decision support systems have been proposed to help in the selection of the best compromise alternatives. One of them is the PROMETHEE (I–IV) method for treating multicriteria problems. This method is known as one of the most efficient but also one of the the easiest in the field. A particularly user–friendly software, called Decision Lab, has been developed in collaboration with the Canadian company Visual Decision (`http://www.visualdecision.com`) to assist all kinds of decision-makers.

The following example, presented in Table 5, is taken from the article [BrMa] distributed with the software, represents complex exact data decision table where each criterion has its own measurement scale.

Unit	**cost**	**target**	**durat.**	**effic.**	**manpower**
min/max	minimize	maximize	maximize	maximize	minimize
rel. weight	12	40	12	22	14
`News`	60	900	22	51	8
`Herald`	30	520	31	13	1
`Panels`	40	650	20	58	2
`Mailing`	92	750	60	36	3
`CMM`	52	780	58	90	1
`NCB`	80	920	4	75	6

TABLE 5. Marketing problem.

Let us suppose that a company intends to advertise its products. Six marketing actions are considered: advertising in the international newspaper `News`, in the newspaper `Herald`, by means of advertising boards in large cities (`Panels`), by personal mailing (`Mailing`), by TV spots on channels `CMM` or `NCB`.

As one can see by looking at the enumeration of the advertising supports, this is an illustrative example, not a real-world one! As shown in Table 5, five evaluation criteria are taken into account: the **cost** expressed in 1.000 US$, the **target** expressed in 10.000 people, the **duration** of the action in days, the **efficiency** expressed on a 0–100 scale and the number of people involved in the action within the company, **manpower**. Some criteria has to be minimized (cost and manpower), the others have to be maximized (target, duration, efficiency). The weights 12, 40, 12, 22, 14 have been initially allocated to criteria. Even for such small problem the best advertising action is not obvious. Indeed no alternative is optimal, i.e. dominates the others, on the five criteria!

	PROMETHEE (Φ)	PM (w)	
	Usual pref. func.	$FlowNorm = 2$	$FlowNorm = 10$
`CMM`	0.51	0.230	0.556
`NCB`	0.26	0.181	0.168
`News`	0.01	0.169	0.118
`Panels`	-0.17	0.152	0.069↓
`Mailing`	-0.24	0.152	0.071↑
`Herald`	-0.36	0.116	0.018

Table 6. Ranking results for PROMETHEE and PM.

In Table 6 the first column (PROMETHEE) gives the results of ranking by Decision Lab 2000 software and the second column (PM) gives the ranks obtained by Potential Method. In PROMETHEE method the shape of value function is 'usual'. All values of $FlowNorm$ less than 4 produce the same ranking (as shown in the table), while greater values of $FlowNorm$ (10 for example) make distinction between `Panels` and `Mailing`.

6. Complex hierarchical structure

In the paper of Yang [Ya] generic decision models and both rule and utility based techniques for transforming assessment information are developed to enhance an Evidential Reasoning (ER) approach for dealing with Multiple Attribute Decision Analysis (MADA) problems of both quantitative and qualitative nature under uncertainties.

A MADA problem may also be modelled using a generalized decision matrix, where an attribute is assessed using a belief structure represented by expectations. An expectation was originally designed to model qualitative assessments with uncertainty in the ER approach developed on the basis of decision theory and the Dempster–Shafer theory of evidence (Yang and Singh [YaSi]). The ER approach has been used to deal with MADA problems in engineering and management, for example motorcycle assessment, Yang [Ya].

6.1. **Cars assessment.** A performance assessment problem of cars is as shown in Table 7. Seven performance attributes are taken into account including four quantitative attributes: Acceleration (seconds from 0 to 60 mph), Braking (feet from 60 mph to 0 mph), Horsepower (hp) and Fuel economy (mpg), and three qualitative ones: Handling, Ride quality and Power train. Six different cars were chosen for this analysis. Note that Acceleration, Braking and Fuel economy are attributes to be minimized and other attributes are to be maximized. Most of the attributes in the table are related to the technical performances of a car.

Suppose the performance of a car is classified into several categories (grades) like 'Top', 'Excellent', 'Good', 'Average', 'Poor' and 'Worst', and equivalence rules are acquired as shown in Table 8.

Performance	Car 1	Car 2	Car 3	Car 4	Car 5	Car 6
Acceleration	8.8	8.0	7.7	8.4	8.0	7.9
Braking	128	124	127	134	135	126
Handling	B	A	B	B-	B+	A
Horsepower	196	152	182	183	138	171
Ride quality	A-	B-	B	B+	B+	A-
Powertrain	B	B+	A	B	A-	A
Fuel economy	20	20	21	20	19	20

Table 7. Performance assessment of cars.

	Worst	Poor	Average	Good	Excellent	Top
C-	1	0	0	0	0	0
C	0.6	0.4	0	0	0	0
C+	0	0.6	0.4	0	0	0
B-	0	0	1	0	0	0
B	0	0	0.4	0.6	0	0
B+	0	0	0	1	0	0
A-	0	0	0	0.6	0.4	0
A	0	0	0	0	0.6	0.4
A+	0	0	0	0	0	1

Table 8. Belief structure for cars assessment.

Cars rating		Car 1	Car 2	Car 3	Car 4	Car 5	Car 6
Yang	Utility	0.6319	0.6448	0.7308	0.5486	0.5178	0.7616
	Ranking	4	3	2	5	6	1
PM (1)	Rank	0.164	0.174	0.183	0.148	0.139	0.193
	Utility	0.4970	0.6814	0.8386	0.1859	0.0000	1.0000
	Ranking	4	3	2	5	6	1
PM (2)	Rank	0.162	0.170	0.194	0.141	0.130	0.202
	Utility	0.5009	0.6054	0.9141	0.1772	0.0000	1.0000
	Ranking	4	3	2	5	6	1

Table 9. Performance rating of cars. Comparison of ER and PM methods.

In Table 9 results of ranking procedure are given for both methods, Yang's ER and Potential Method. Results in the row PM (1) are obtained directly from data in Table 7, with value of $FlowNorm$ set to 1. To apply ER algorithm, data from Table 7 had to be transformed to 'overall degrees of belief' given in [Ya, Table 2, p. 49] called 'transformed distributed assessment of executive cars'. We also calculated ranks from this transformed table and results are given in row PM (2). In his article, Yang gave comparative results with several other methods: MAUF method of Hwang and Yoon [HwYo], Saat'y AHP [Sa], Belton's procedure [BeGe] and Jonson's procedure [Jo]. All of them gave the same ranking order except Saaty's eigenvalue method which reversed the positions of Car 1 and Car 2. All of them placed Car 1 and Car 2 relatively close which may be the cause of reversing their position in Saaty's approach.

The ER method in some cases requires further analysis, while the PM is much more straightforward. These analysis can produce further errors which are unavoidable in all decision making calculations. This means that any inconsistence produced by subjective nature of the input data can get enlarged even more than it is usual.

Also, "overall degrees of belief" in ER approach are not identical to the original input (in terms of decision making). This can easily be seen in the difference between rows PM(1) and PM(2).

On the other hand, PM adapts the input much more transparently (with *FlowNorm*), reducing both errors and possibility of abuse to a minimal level.

6.2. **Incomplete data. Motorcycle assessment.** Much more complicated example is motorcycle assessment problem based on 29 attributes of a hierarchy which is taken from [Ya, Table 7, p. 52]. Input data is rewritten in Tables 11 and 12 which present original data transformed to overall degrees of belief. Relative weights of the same group of attributes are shown in brackets. For criterion 'Fuel consumption' four consumption values are given: in/out urban area in winter/summer, with relative weights 1, 1, 1, 1.

Complexity of the problem arises from mixture of precise and imprecise data as well as because of the missing data. Missing data is of two kinds in its nature: *non-existing data* and *incomplete belief*. For instance, the fuel consumption for Yamaha in urban area in winter is unknown and engine responsiveness of Yamaha has total belief degree 0.9.

In ER approach the qualitative attributes are all assessed on the basis of the same five evaluation grades which are defined as poor, indifferent, average, good and excellent and abbreviated by P, I, A, G and E, respectively. The overall assessment of a motorcycle is also based on this set of grades.

In PM approach we fixed $FlowNorm = 0.5$ for criteria: range, displace, speed and $FlowNorm = -0.5$ for price. All qualitative attributes P, I, A, G and E have $FlowNorm = 4$ while all criteria for fuel consumption have $FlowNorm = 0.5$. Those values were chosen as a result of experimentation but don't have strong influence on final ranking. It is only important to maintain the same $FlowNorm$ value for major criteria.

Method		Kawasaki	Yamaha	Honda 3	BMW
Yang (ER)	Average Utility	0.6232	0.666	0.7118	0.5847
	Ranking	3	2	1	4
PM	Rank	0.251	0.253	0.260	0.235
	Ranking	3	2	1	4

TABLE 10. Comparative results for motorcycle assessment.

Table 10 gives comparative results of Yang's ER method and PM. While constructing consensus flow in PM we used formula (2), without attempt to make any kind of reconstruction of missing data. We repeated the calculation with data recovering using homogeneous distribution with minor differences in final numerical values of ranks.

Attribute	Kawasaki	Yamaha	Honda	BMW
Price, pounds (9)	6499	5199	6199	8220
Displacement, cc (5)	1052	1188	998	987
Range, miles (7)	175	160	170	200
Top speed, mph (7)	160	155	160	145
Engine (14)				
Responsiveness, (0.2)	(E, 0.8)	(G, 0.3) (E, 0.6)	(G, 1.0)	(I, 1.0)
Fuel consumption, mpg (0.4)	(32, 0.25) (36, 0.25) (41, 0.25) (43, 0.25)	(28, 0.25) (34, 0.25) (38, 0.25)	(31, 0.25) (35, 0.25) (39, 0.25) (43, 0.25)	(35, 0.25) (39, 0.25) (46, 0.25) (48, 0.25)
Quietness (0.1)	(I, 0.5) (A, 0.5)	(A, 1.0)	(G, 0.5) (E, 0.3)	
Vibration (0.1)	(G, 1.0)	(I, 1.0)	(G, 0.5) (E, 0.5)	(P, 1.0)
Starting (0.2)	(G, 1.0)	(A, 0.6) (G, 0.3)	(G, 1.0)	(A, 1.0)

TABLE 11. Data for motorcycle selection problem. Part A.

7. Conclusion

In this article we have proposed an extension to Potential Method which allows decision makers to apply PM to completely new range of problems. As we have seen, this extension is very natural and leads to virtually no changes in the approach, although the input was radically changed (from pairwise comparisons to the exact data).

Further, we gave a clue on a way to handle incomplete data, which is currently undergoing research.

To show that PM is applicable for this sort of problems, we have compared it to the ER and PROMETHEE with excellent results.

Finally, the fine-tuning tool of the PM - $FlowNorm$ - proved to be very natural, while also extremely powerful in handling different scales. Also, the transparency of such approach should not be overseen, for the reasons explained in the article.

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Attribute	Kawasaki	Yamaha	Honda	BMW
Operation (7)				
Handling (0.5)				
Steering (0.3)	(E, 0.9)	(G, 1.0)	(A, 1.0)	(A, 0.6)
Bumpy bends (0.1)	(A, 0.5) (G, 0.5)	(G, 1.0)	(G, 0.8) (E, 0.1)	(P, 0.5) (I, 0.5)
Manoeuvrability (0.4)	(A, 1.0)	(E, 0.9)		(P, 1.0)
Top speed stability (0.3)	(E, 1.0)	(G, 1.0)	(G, 1.0)	(G, 0.6) (E, 0.4)
Transmission (0.167)				
Clutch operation (0.5)	(A, 0.8)	(G, 1.0)	(E, 0.85)	(I, 0.2) (A, 0.8)
Gearbox operation (0.5)	(A, 0.5) (G, 0.5)	(I, 0.5) (A, 0.5)	(E, 1.0)	(P, 1.0)
Brakes (0.333)				
Stopping power (0.4)	(G, 1.0)	(A, 0.3) (G, 0.6)	(G, 1.0)	
Braking stability (0.3)	(G, 0.5) (E, 0.5)	(G, 1.0)	(A, 0.5) (G, 0.5)	(E, 1.0)
Feel at control (0.3)	(P, 1.0)	(G, 0.5) (E, 0.5)	(G, 1.0)	(G, 0.5) (E, 0.5)
General (14)				
Quality of finish (0.4)	(P, 0.5) (I, 0.5)	(G, 1.0)	(E, 1.0)	(G, 0.5) (E, 0.5)
Seat comfort (0.3)	(G, 1.0)	(G, 0.5) (E, 0.5)	(G, 1.0)	(E, 1.0)
Headlight (0.1)	(G, 1.0)	(A, 1.0)	(E, 1.0)	(G, 0.5) (E, 0.5)
Mirrors (0.1)	(A, 0.5) (G, 0.5)	(G, 0.5) (E, 0.5)	(E, 1.0)	(G, 1.0)
Horn (0.1)		(G, 1.0)	(G, 0.5) (E, 0.5)	(E, 1.0)

TABLE 12. Data for motorcycle selection problem. Part B.

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